

Looking for meson molecules in B decays

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We discuss the possibility of observing a loosely bound molecular state in a B three-body hadronic decay. In particular we use the QCD sum rule approach to study a $\eta' - \pi$ molecular current. We consider an isovector-scalar $I^G J^{PC} = 1^- 0^{++}$ molecular current and we use the two-point and three-point functions to study the mass and decay width of such state. We consider the contributions of condensates up to dimension six and we work at leading order in α_s . We obtain a mass around 1.1 GeV, consistent with a loosely bound state, and a $\eta' - \pi \rightarrow K^+ K^-$ decay width around 10 MeV.

I. INTRODUCTION

One of the outstanding open questions in hadron physics is: are there meson-meson bound states? The same mechanism of meson exchange that binds the deuteron could also in principle bind two mesons. The interest in this subject was renewed by the discovery of the new charmonium states. Since their first appearance, some of them were considered to be meson molecules. These states have already been discussed in some reviews [1–3].

In this note we discuss how to look for molecules at the LHCb taking advantage of the unprecedented high statistics.

We can look for meson molecules in three-body hadronic B decays. Since the phase space is large we can even try to use directly the Dalitz diagram, which extends up to large values of the variables s_{12} and s_{23} . All the known normal quark-antiquark intermediate resonant states, leave an imprint in the Dalitz plot, which is directly related with the quantum numbers of the states and lead to the identification of the state. Examples are: a continuous straight line, in the case of scalar states, a line with a hole, in the case of vector states, or a line with two holes, in the case of tensor states.

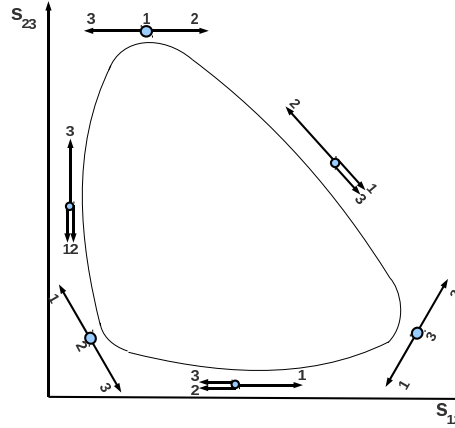


FIG. 1: Dalitz plot of a three-body B decay. The small drawings illustrate the different kinematical configurations.

A sketch of a Dalitz plot for a three-body meson decay is shown in Fig. I, where for each invariant parameter s_{12} or s_{23} it is shown the relative momentum of each one of the two particles 1, 2 or 2, 3. Let us consider the case that the particles 1 and 2 are pions coming from the ρ meson decay. Of course this decay should produce a line parallel to the s_{23} axis in the point $s_{12} = m_\rho^2$. However, since the pions coming from the ρ meson decay must have one unit of angular momentum, they cannot go both to the same direction. Therefore, no pions could be seen in the region where the relative momentum between them is small. From Fig.I one can see that this region is just in the middle

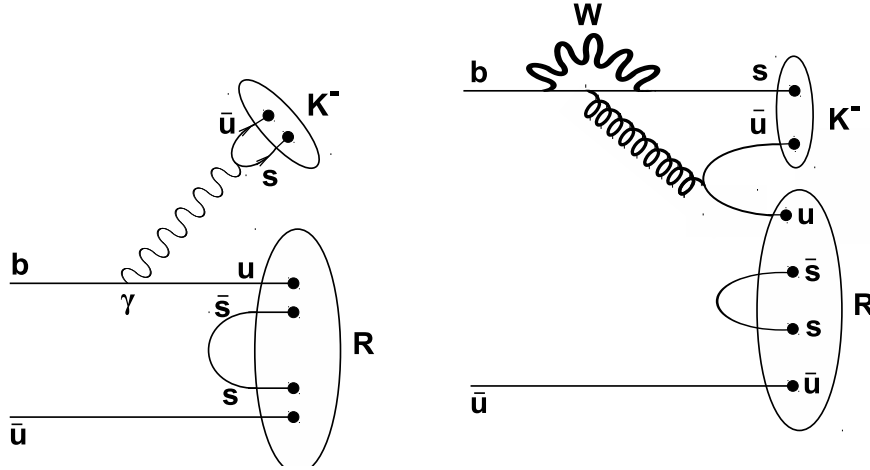


FIG. 2: The two relevant diagrams for the $B^- \rightarrow K^+ K^- K^-$, through the resonance R .

of the line parallel to the s_{23} axis. Therefore, a line characterizing a vector resonance state must have a hole in the middle, as mentioned above. Now imagine that the resonant state is a loosely bound molecular state of the particles 1, 2. A loosely bound molecular state can only exist when the relative momentum between the two mesons in the molecule is small. In this case one has exactly the opposite situation than the one discussed before: there will be no signal in the Dalitz plot unless the two mesons in the molecular state go in the same direction. Or, in other words, one expects a small line parallel to the s_{23} axis in the middle of the Dalitz plot, approximately in the region where there is a hole in the line characterizing a vector resonance.

The final particles observed in the three-body B decays are pions and kaons. Therefore, to observe a molecular state in the Dalitz plot for a three-body B decay, this molecular state must decay into pions and/or kaons. Let us consider a $\eta' - \pi$ loosely bound molecule with the quark content $\bar{u}u\bar{s}s$. This resonant state, hereafter called R , is especially interesting because its mass, m_R , should be approximately given by:

$$m_R \sim m_{\eta'} + m_\pi = 958 + 138 = 1096 \text{ GeV} \quad (1)$$

and, therefore, it is quite visible in the B decay Dalitz plot. Since for S -wave this molecule has $I^G J^{PC} = 1^- 0^{++}$, it can not decay into $\pi^+\pi^-$, but it will decay into K^+K^- . In particular, there are already data for $B^- \rightarrow K^+K^-K^-$, $B^- \rightarrow K^+K^-\pi^-$, $B^- \rightarrow \pi^+\pi^-\pi^-$ and $B^- \rightarrow \pi^+\pi^-K^-$ [4]. Only in the first two of these cases the decay could go through the resonant state R , as illustrated in Figs. 2 and 3. For these cases, a small line with $\sqrt{s_{12}} \sim 1.1 \text{ GeV}$ parallel to the s_{23} axis should be seen in the Dalitz plot, in the region where the two particles η' and π have a small relative momentum. This signal should be very different from all other established resonant states decaying into K^+K^- , like the a_0 for instance, and should be only seen in the channels $B^- \rightarrow K^+K^-K^-$ and $B^- \rightarrow K^+K^-\pi^-$. Of course, the figure is very qualitative and it is not possible to say how large is the line segment around the indicated position. However, the observation of this structure in the Dalitz plot of the two mentioned B decays, and not in the others, would represent a strong evidence of the formation of this molecular state. The observation of a line that only appears in a certain piece of the s_{23} axis with a fixed value of s_{12} , and only for the decays $B^- \rightarrow K^+K^-K^-$ and $B^- \rightarrow K^+K^-\pi^-$, could be interpreted as the existence of a weakly bound molecular state.

II. THE $\eta' - \pi$ SCALAR MOLECULE

A. Mass

In a previous work [5] we have investigated the possibility that the light scalar states could be interpreted as tetraquark states. Now we perform a complementary investigation, trying to understand a possible η'/π meson molecular state in the QCD sum rule (QCDSR) framework [6–8].

The QCDSR approach is based on the correlator of hadronic currents. A generic two-point correlation function is given by

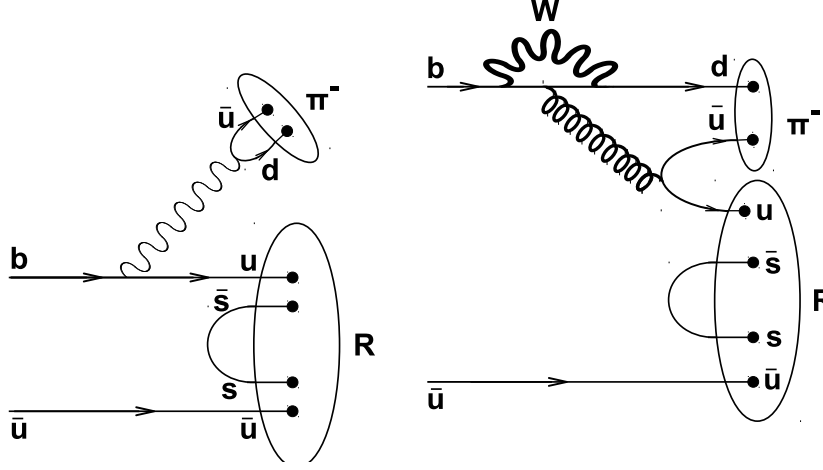


FIG. 3: The two relevant diagrams for the $B^- \rightarrow K^+ K^- \pi^-$ decay, through the resonance R .

$$\Pi(q) = i \int d^4x e^{iq \cdot x} \langle 0 | T[j(x) j^\dagger(0)] | 0 \rangle, \quad (2)$$

where the local current $j(x)$ contains all the information about the hadron of interest, like quantum numbers, quarks content and so on. A molecular current can be constructed from the mesonic currents that describe the two mesons in the molecule. In the case of scalar $\eta' - \pi$ state a possible current is:

$$j = \left(\frac{\bar{u}_i \gamma_5 u_i - \bar{d}_i \gamma_5 d_i}{\sqrt{2}} \right) \left[\sin \theta \left(\frac{\bar{u}_j \gamma_5 u_j + \bar{d}_j \gamma_5 d_j}{\sqrt{2}} \right) + \cos \theta (\bar{s}_j \gamma_5 s_j) \right], \quad (3)$$

where i, j are color indices, u, d, s are the up down and strange quark fields respectively, and the mixing angle, θ , in the η' current is $\theta \sim 40^\circ$ [9–11]. In this work we use $\theta = 40^\circ$.

In general, there is no one to one correspondence between the current and the state, since a molecular current can be rewritten in terms of a sum over tetraquark type currents through a Fierz transformation. However, as shown in [2], if the physical state is a molecular state, it would be better to choose a molecular type of current so that it has a large overlap with the physical state. In any case, it is very important to notice that since the current in Eq. (2) is local, it does not represent an extended object, with two mesons separated in space, but rather a very compact object with two singlet quark-antiquark pairs.

The coupling of the scalar resonance R , to the scalar current j , can be parametrized in terms of a parameter λ as:

$$\langle 0 | j | R \rangle = \lambda. \quad (4)$$

In the QCD side evaluation of the correlator function in Eq.(2) we work at leading order and consider condensates up to dimension six. We deal with the strange quark as a light one and consider the diagrams up to order m_s . We neglect the terms proportional to m_u and m_d . In the phenomenological side we consider the usual pole plus continuum contribution. Therefore, we introduce the continuum threshold parameter s_0 [12]. In the $SU(2)$ limit the quarks u and d are degenerate and we consider the u -quark condensate equal to the d -quark condensate, wich we call $\langle \bar{q}q \rangle$. After doing a Borel tranform in both sides of the calculation the sum rule is given by:

$$\begin{aligned} \lambda^2 e^{-m_R^2/M^2} &= 3 \frac{M^{10} E_4}{2^{13} 5 \pi^6} (12 + \sin^2 \theta) - \frac{m_s \langle \bar{s}s \rangle M^6 E_2}{2^7 \pi^4} \cos^2 \theta + \frac{\langle g^2 G^2 \rangle M^6 E_2}{2^{13} \pi^6} (4 - \sin^2 \theta) \\ &- \frac{m_s \langle \bar{s}g\sigma G s \rangle}{2^7 \pi^4} M^4 E_1 \cos^2 \theta (3.5 - 3 \ln(M^2/\Lambda_{QCD}^2)) \\ &+ \frac{M^4 E_1}{2^6 \pi^2} (\langle \bar{q}q \rangle^2 (1 + 3 \cos^2 \theta) + 2 \langle \bar{s}s \rangle^2 \sin^2 \theta), \end{aligned} \quad (5)$$

where

$$E_n \equiv 1 - e^{-s_0/M^2} \sum_{k=0}^n \left(\frac{s_0}{M^2} \right)^k \frac{1}{k!}, \quad (6)$$

which accounts for the continuum contribution.

In the numerical analysis of the sum rules, the values used for the quark masses and condensates are [13, 14]: $m_s = 0.13$ GeV, $\langle \bar{q}q \rangle = -(0.23)^3$ GeV³, $\langle \bar{s}s \rangle = 0.8\langle \bar{q}q \rangle$, $\langle \bar{q}g\sigma.Gq \rangle = m_0^2\langle \bar{q}q \rangle$ with [8] $m_0^2 = 0.8$ GeV² and $\langle g^2 G^2 \rangle = 0.88$ GeV⁴.

In Fig. 4 we show the OPE convergence of the sum rule in Eq. (5). From this figure we see that the convergence is reasonable for $M^2 > 1.2$ GeV² and very good for $M^2 > 1.5$ GeV². However, as in the case of the light scalars [14], there is no pole dominance for these values of M^2 . This result could be interpreted in two different ways: i) it could indicate that this state does not exist, or ii) it could indicate that this state is not clearly separated from the continuum. The second interpretation can be applied to very broad states, as the light scalars σ and κ , since their widths are as large as the difference between their masses and the continuum threshold. In what follows we stick to the interpretation ii).

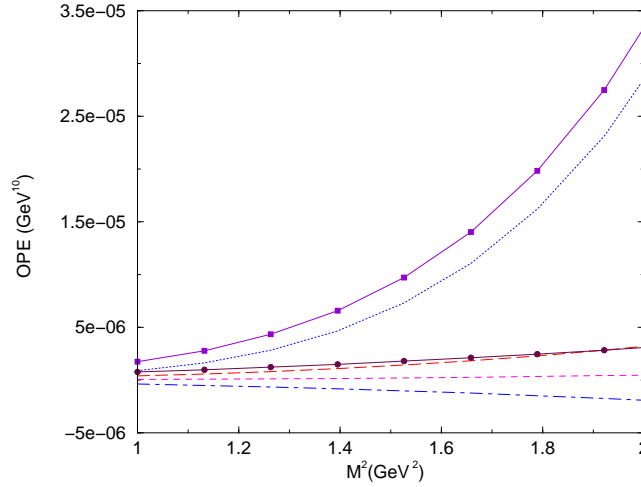


FIG. 4: The OPE convergence in the region $1.0 \leq M^2 \leq 2.0$ GeV² for $\sqrt{s_0} = 1.5$ GeV. The dotted, dashed, long-dashed, dot-dashed, solid with circles and solid white squares lines give the perturbative, quark condensate, gluon condensate, mixed condensate, four-quark condensate and total contributions respectively.

In order to extract the mass m_R without knowing the value of the constant λ , we take the derivative of Eq. (5) with respect to $1/M^2$ and divide the result by Eq. (5). In Fig. 5, we show the resonance mass as a function of M^2 for different values of $\sqrt{s_0}$. We limit ourselves to the region $M^2 > 1.2$ GeV² where the curves are more stable and where the OPE convergence is better. Averaging the mass over all this region we find:

$$m_R = (1.15 \pm 0.10) \text{ GeV}, \quad (7)$$

which is compatible with the experimental threshold in Eq.(1). Having the mass, we can also evaluate the value of the parameter λ that gives the coupling between the state and the current. We obtain:

$$\lambda = (1.39 \pm 0.27) \times 10^{-3} \text{ GeV}^5. \quad (8)$$

B. Decay width

In order to study the RK^+K^- vertex associated with the $R \rightarrow K^+K^-$ decay, we consider the three-point function

$$T_{\mu\nu}(p, p', q) = \int d^4x d^4y e^{i.p'.x} e^{iq.y} \langle 0 | T \{ j_{5\mu}^{K^+}(x) j_{5\nu}^{K^-}(y) j^\dagger(0) \} | 0 \rangle, \quad (9)$$

where $p = p' + q$, j is given in Eq. (3) and we use the axial currents for the kaons:

$$j_{5\mu}^{K^+} = \bar{s}_a \gamma_\mu \gamma_5 u_a, \quad j_{5\mu}^{K^-} = \bar{u}_a \gamma_\mu \gamma_5 s_a. \quad (10)$$

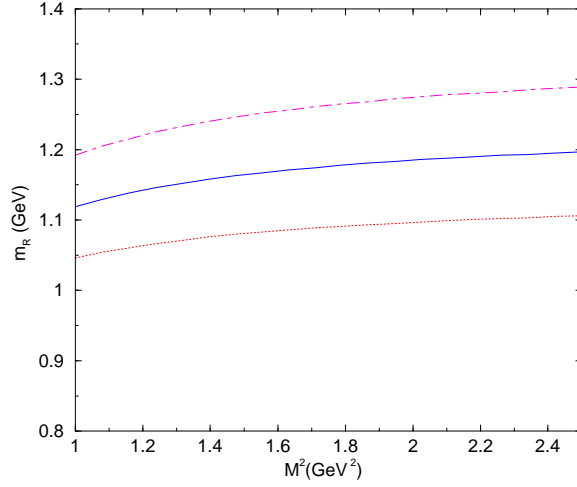


FIG. 5: The resonance mass as a function of the sum rule parameter (M^2) for different values of the continuum threshold: $\sqrt{s_0} = 1.4$ GeV (dotted line), $\sqrt{s_0} = 1.5$ GeV (solid line) and $\sqrt{s_0} = 1.6$ GeV (dot-dashed line).

To evaluate the phenomenological side we insert intermediate states for K^+ , K^- and R , and we use the definitions in Eqs. (4) and (11) below:

$$\langle 0 | j_{5\mu}^K | K(p) \rangle = i p_\mu F_K. \quad (11)$$

We obtain the following relation:

$$T_{\mu\nu}^{phen}(p, p', q) = \frac{F_K^2 \lambda}{(M_R^2 - p^2)(m_K^2 - p'^2)(m_K^2 - q^2)} g_{RKK} p'_\mu q_\nu + \text{higher resonances}, \quad (12)$$

where the coupling constant g_{RKK} is defined by the matrix element:

$$\langle K(p') K(q) | R(p) \rangle = g_{RKK}. \quad (13)$$

Here we follow refs. [5, 15] and work at the kaon pole, as suggested in [7] for the nucleon-pion coupling constant. This method was also applied to the nucleon-kaon-hyperon coupling [16, 17], to the $D^* - D - \pi$ coupling [18, 19] and to the $J/\psi - \pi$ cross section [20]. It consists in neglecting the kaon mass in the denominator of Eq. (12) in the term $1/(m_K^2 - q^2)$, and working at $q^2 = 0$. In the QCD side one singles out the leading terms in the operator product expansion of Eq.(9) that match the $1/q^2$ term. Up to dimension six only the diagrams proportional to the quark condensate times m_s and the four-quark condensate contribute. Making a single Borel transform to both $-p^2 = -p'^2 \rightarrow M^2$ we get:

$$g_{RK^+K^-} \frac{\lambda F_K^2}{m_R^2 - m_K^2} (e^{-m_K^2/M^2} - e^{-m_R^2/M^2}) = \frac{\sqrt{2} \cos \theta}{8} \left(\frac{(\langle \bar{q}q \rangle + \langle \bar{s}s \rangle)^2}{3} + \frac{m_s}{8\pi^2} \left(\langle \bar{q}q \rangle - \frac{\langle \bar{s}s \rangle}{3} \right) M^2 \left(1 - e^{-s_0^K/M^2} \right) \right), \quad (14)$$

where $s_0^K = (1.0 \pm 0.1) \text{ GeV}^2$, is the continuum threshold for the kaon.

As discussed in ref. [21], the problem of doing a single Borel transformation, in a three-point function sum rule, is the fact that terms associated with the pole-continuum transitions are not suppressed. However, as shown in [21], the pole-continuum transition term has a different behavior, as a function of the Borel mass, as compared with the double pole contribution: it grows with M^2 . Therefore, the pole-continuum contribution can be taken into account through the introduction of a parameter A in the phenomenological side of the sum rule in Eq. (14), by making the substitution $g_{RK^+K^-} \rightarrow g_{RK^+K^-} + AM^2$ [5, 17, 18, 20].

Using $F_K = 160 \text{ MeV}$, $m_K = 490 \text{ MeV}$, $m_R = 1.15 \text{ GeV}$ and the parameter λ given by the sum rule in Eq. (5) we show, in Fig. 6, the QCDSR results for the vertex coupling constant, for different values of s_0 and s_0^K in the interval given above. We see that, in the Borel range used for the two-point function, the QCDSR results do have a linear behaviour as a function of the Borel mass. Fitting the QCDSR results by a linear form: $g_{RK^+K^-} + AM^2$ (which is

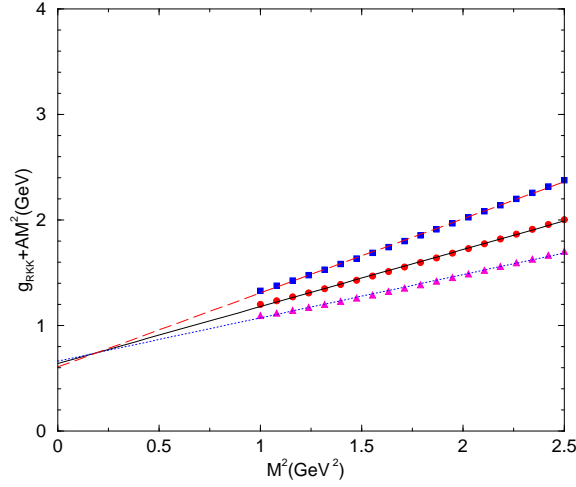


FIG. 6: The QCDSDR result for the coupling constant $g_{RK^+K^-}$ as a function of the sum rule parameter M^2 for different values of s_0 and s_0^K (circles, triangles and squares for $\sqrt{s_0} = 1.5$ GeV, $s_0^K = 1.0$ GeV²; $\sqrt{s_0} = 1.6$ GeV, $s_0^K = 1.1$ GeV² and $\sqrt{s_0} = 1.4$ GeV, $s_0^K = 0.9$ GeV² respectively). The solid, dotted and dashed lines give the linear fit to the QCDSDR results.

also shown in Fig. 6), the coupling can be obtained by extrapolating the fit to $M^2 = 0$. In the limits of the continuum thresholds mentioned above and taking into account the uncertainties in m_R given in Eq. (7) we obtain:

$$g_{RK^+K^-} = (0.63 \pm 0.06) \text{ GeV}. \quad (15)$$

The decay width of $R \rightarrow K^+K^-$ is given in terms of the hadronic coupling $g_{RK^+K^-}$ as:

$$\Gamma(R \rightarrow K^+K^-) = \frac{1}{16\pi m_R^3} g_{RK^+K^-}^2 \sqrt{\lambda(m_R^2, m_K^2, m_K^2)}, \quad (16)$$

where $\lambda(m_R^2, m_K^2, m_K^2) = m_R^4 + m_K^4 + m_K^4 - 2m_R^2 m_K^2 - 2m_R^2 m_K^2 - 2m_K^2 m_K^2 = m_R^2(m_R^2 - 4m_K^2)$. Therefore, we get:

$$\Gamma(R \rightarrow K^+K^-) = (11.4 \pm 2.2) \text{ MeV}. \quad (17)$$

Of course this is not the total width of the $\eta'\pi$ molecule, since it can also decay into $\eta - \pi$ with a much bigger phase space. However, in the B decays discussed here, only the channel $R \rightarrow K^+K^-$ can be observed.

The errors quoted above come directly from the uncertainty in the determination of the continuum threshold parameters, s_0 . According to our previous experience, they are the main source of uncertainty in the method. For a detailed analysis of the uncertainty associated to other parameters used in QCDSDR we refer the reader to Refs. [2] and [19].

III. CONCLUSION

We have proposed that a loosely bound molecular state should leave a particular signal in the Dalitz plot. A loosely bound molecular state, of the particles 1, 2, can only exist when the relative momentum between these two particles is small. Therefore, we expect to observe a short line parallel to the s_{23} axis in the middle of the Dalitz plot, approximately in the region where there is a hole in the line characterizing a vector resonance (see Fig. I). This signal is different from any signal characterizing the normal quark-antiquark mesons, and could be used to identify the existence of loosely bound molecular states.

In the case of three-body B decays, the final particles observed are pions and kaons. Therefore, to observe a molecular state in the Dalitz plot for a three-body B decay, this molecular state must decay into pions and/or kaons. We have considered a $\eta' - \pi$ molecular state. If this state exists as a loosely bound state, its mass should be close to the $\eta' - \pi$ threshold: ~ 1.1 GeV, quite visible in the B decay Dalitz plot. Since for a S -wave this molecule has $I^G J^{PC} = 1^- 0^{++}$, it can not decay into $\pi^+\pi^-$, but it will decay into K^+K^- . Therefore, the observation of a small line with $\sqrt{s_{12}} \sim 1.1$ GeV parallel to the s_{23} axis in the Dalitz plot for the $B^- \rightarrow K^+K^-K^-$ and $B^- \rightarrow K^+K^-\pi^-$ decays, with negative observation in the Dalitz plot for the $B^- \rightarrow \pi^+\pi^-K^-$ and $B^- \rightarrow \pi^+\pi^-\pi^-$ decays would definitively indicate the existence of the $\eta' - \pi$ molecular state.

We have used QCD sum rules to study the mass and the decay width, of a $\eta' - \pi$ molecular current, using two-point and three-point functions respectively. We have considered diagrams up to dimension six in both cases. We found a mass a slightly larger than the $\eta' - \pi$ threshold, indicating the possibility of a loosely bound molecular state. We obtained a small width for the $\eta' - \pi \rightarrow K^+ K^-$ decay around 10 MeV. With these informations should be possible to experimentally identify this state in the $B^- \rightarrow K^+ K^- K^-$ and $B^- \rightarrow K^+ K^- \pi^-$ Dalitz plots, if it exists.

The method for the identification of resonances (or bound states) discussed here could be applied to other cases. A straightforward extension of our work could be done to the $\eta' - \pi$ with quantum numbers $J^{PC} = 1^{-+}$. This exotic state has been recently searched for by the CLEO [22] and COMPASS [23] collaborations.

Acknowledgments

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